A Brief Introduction to System Dynamics Modelling

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### Glossary of System Dynamics Terminology

<table>
<thead>
<tr>
<th>Term</th>
<th>Description</th>
<th>Symbol</th>
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<tbody>
<tr>
<td><strong>All other things being equal</strong></td>
<td>When reading causal relationships in a model of a feedback loop it is necessary to view the relationship to the exclusion of other non-connected variables. Thus the <em>partial derivative</em> in engineering and the term <em>ceteris paribus</em> in management are linked. See Link Polarity.</td>
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<tr>
<td><strong>Balancing Feedback</strong></td>
<td>An arrangement of feedback that leads to balancing or goal-seeking behaviour. Also known as negative feedback</td>
<td><img src="image" alt="Balancing feedback" /></td>
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<tr>
<td><strong>Bounded Rationality</strong></td>
<td>Term originating from work of Herb Simon. The limited ability of human intuition (comprising information, time, and computational resource) to make rational decisions in complex situations.</td>
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<tr>
<td><strong>Causal Loop</strong></td>
<td>A pattern of mutually causal influences i.e. A causes B, B causes C, ... X causes A. This loop is also known as a feedback loop and can be balancing or reinforcing. Hence Causal Loop Diagram, a diagram of such feedback loops.</td>
<td><img src="image" alt="Causal Loop Diagram" /></td>
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<tr>
<td><strong>Delays</strong></td>
<td>Feedback is never instantaneous and may range from fractions of seconds to years. Delays are another contributing factor to creating complex dynamics. Delays make goal seeking difficult to achieve.</td>
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<tr>
<td><strong>Endogenous</strong></td>
<td>The property of originating from within. In the case of systems models, all the complex behaviour we observe can be accounted for by the structure of the system model alone.</td>
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<tr>
<td><strong>Exogenous</strong></td>
<td>The property of originating from without. In the case of systems models, complex behaviour is determined by factors originating from outside of the model.</td>
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<tr>
<td><strong>Feedback</strong></td>
<td>Information about the state of a Stock that is used by itself, or in combination with other information, to modify a Flow. It is feedback, in various combinations of balancing and reinforcing patterns, that leads to complex dynamic behaviour in a system.</td>
<td><img src="image" alt="Feedback" /></td>
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<tr>
<td><strong>Flow</strong></td>
<td>The flow of something into or out of a Stock. The thing flowing could be more or less anything that is measurable; money, vehicles, CO₂, etc., through to more conceptual things like perceptions and behaviours e.g. confidence. Will normally be measured as a rate e.g. vehicles/year</td>
<td><img src="image" alt="Flow" /></td>
</tr>
</tbody>
</table>
Goal Seeking  The property of balancing or negative feedback to control a stock towards a particular value. The task of filling a glass to a mark is an example of goal seeking, balancing feedback.

Information  The fundamental characteristic of feedback either in the Shannon sense, or physical e.g. temperature, pressure etc. This broad definition allows for system dynamics models to be a mix of social and physical things.

Link Polarity  Causal influences can either be +ve or –ve. If A causes B with +ve influence then, all other things being equal, as A increases (decreases) B increases (decreases) i.e. they move in the same direction. If X causes Y with -ve influence then, all other things being equal, as X increases (decreases) Y decreases (increases) i.e. they move in the opposite direction.

Negative feedback  Another term for balancing feedback.

Positive feedback  Another term for reinforcing feedback.

Reinforcing Feedback  A pattern of feedback that leads to growth in the state of a system. This will continue until the system hits some exogenous factor that limits the growth e.g. exhaustion of a material quantity outside the system of interest.

Stock  Something that accumulates a flow, or from which a flow originates. Since an accumulation of a flow is integration, the units of a Stock are generally not measured as rates, i.e. not with respect to time. Examples include a budget, assets (e.g. lengths of road, landing slots), CO₂ in the atmosphere, stored energy, etc. Stocks frequently represent the things in a system that are of interest and can be thought of as the ‘state’ of a system.

System Boundary  Explicitly represented by a cloud symbol. The source or sink of a flow is outside the concern of the system of interest.
1. Introduction to System Dynamics Modelling

System Dynamics modelling originated from pioneering work at MIT in the 1950s by Jay Forrester. His training as an engineer and experience in feedback control systems during the second world war and his subsequent interest in management problems and access to the first campus computers at MIT led to the first significant work in understanding the dynamics of supply chains and a complete model-based theory to explain the bullwhip effect\(^1\). This was first published in HBR (Forrester, 1958) and the field of study launched as Industrial Dynamics. Since then, the System Dynamics modelling community has grown to be a thriving established academic field of study. The annual International Conference of the System Dynamics Society will be holding its 32\(^{nd}\) meeting in the summer of 2014 and these events which alternate between the US and Europe or Asia frequently attract more than 500 participants. They present the application of system dynamics modelling to a vast range of application domains. The author has contributed to recent work at Defra on the use of System Dynamics modelling in policy development (Freeman, Yearworth, Angulo, & Quested, 2013; Freeman, Yearworth, & Cherruault, 2013; Freeman, Yearworth, Jones, & Cherruault, 2013) and with the Transport Systems Catapult on Group Model Building (Yearworth, 2014a, 2014b). Recent journal reviewing experience and literature reviews provide evidence that the approach is being used in research published in journals such as Energy Policy, Journal of Cleaner Production, and the Journal of Industrial Ecology. This indicates a wider acceptance of the need to explore the dynamic complexity of systems using a formal modelling approach.

A useful introduction to System Dynamics modelling can be found in (Sterman, 2000). Modelling software to support the approach is available from Ventana Systems Inc\(^2\) and isee systems amongst others.

2. Key Concepts in System Dynamics

What is System Dynamics modelling? Quoting Peter Senge, it is

“a discipline for seeing wholes. It is a framework for seeing interrelationships rather than things, for seeing patterns of change rather than static snapshots...systems thinking is a discipline for seeing the ‘structures’ that underlie complex situations, and for discerning high and low leverage change.” (Senge, 1990)

The key word here is structure; dynamic behaviour is a consequence of system structure and information feedback. System Dynamics modelling offers an approach to explicit representation of the structure that leads to the dynamic complexity we see in the world. Thus it is possible that socio-technical systems can be modelled and studied as information feedback control systems. The basic structural mechanism in System Dynamics modelling is the notion of feedback loops of mutual causality i.e. A causes B, B causes C, ... causes A; these are known as causal loops. Within this basic schema there are only two forms of feedback:

- Negative feedback, also known as balancing feedback, which leads to goal seeking or control behaviour in a system
- Positive feedback, also known as reinforcing feedback, which leads to unlimited growth until bounded by exogenous factors

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1 And is often called the Forrester effect in his honour.
2 The models in this report were developed using Vensim Pro from Ventana Systems Inc. Versions of the software are available for download from https://vensim.com/vensim-software/
Compounding this feedback and leading to greater complexity is the fact that causality is never an instantaneous effect and is subject to delays that can range from practically instantaneously, in the case of computer mediated information flows, to multiple decades, in the case of large infrastructure projects or effects of anthropogenic climate change. It is the combination of multiple feedback loops and delays in causal relationships that leads to the dynamic complexity we observe in systems. At a gross level, either the positive or negative feedback dominates or we observe damped oscillation, limit cycles, or even chaotic behaviour in a system consisting of multiple feedback loops. Note that this is not detail complexity, system structure can be quite simple yet still produce complex dynamic behaviour. Herb Simon’s work cautions us to the limitations of human perceptual abilities alone to understand dynamic complexity and introduced the notion of bounded rationality in human decision making (Simon, 1991, 1997). Some of the characteristics of dynamic complexity that cause problems with policy interventions are listed in Table 1. This leads us to the need for computer supported modelling techniques to explore possible dynamic behaviour through simulations.

<table>
<thead>
<tr>
<th>Property</th>
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<tbody>
<tr>
<td>Dynamic</td>
<td>Change in the system takes place at many different timescales</td>
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<tr>
<td>Self-organising</td>
<td>The dynamics of a system arise spontaneously</td>
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<tr>
<td>Non-linear</td>
<td>Effects are rarely proportional to cause. Also, the basic physics of the system need to be taken into account</td>
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<tr>
<td>Adaptive</td>
<td>The behaviours of the agents in a complex system will change over time as they learn</td>
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<tr>
<td>Policy resistant</td>
<td>Complexity mitigates against understanding, therefore actions taken without understanding lead to failure or unintended consequences</td>
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<tr>
<td>Counterintuitive</td>
<td>In a complex system cause and effect may be separated by considerable time and space leading to behaviour that is difficult for human agents to follow</td>
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<tr>
<td>History dependent</td>
<td>Some actions may be irreversible, taking one action may preclude taking others</td>
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Table 1. Characteristics of dynamic complexity adapted from (Sterman, 2000).

3. Assumptions

System Dynamics modelling is predicated on a number of assumptions

- A structural account of system dynamics expressed as patterns of feedback is enough to explain complex system behaviour for successful interventions to be designed
- There exists the capability to elicit a structural account of dynamic complexity expressed as a system dynamics model through a process of direct or indirect engagement with domain experts through the following approaches:
  i) Interviews,
  ii) Group model building workshops, and/or
  iii) Analysis of written documentation
- Models can be parameterised for simulation, which can then reproduce currently observed and historical behaviour; so called reference modes of behaviour. They can then be used for
prediction by running a simulation forward in time. The closeness of fit of the model behaviour to data in the former case providing some degree of confidence in the latter

- Insight gained from modelling provides an understanding of where it may be possible to intervene in the system and the effects of such interventions also modelled as a check. Intervention can be through
  
  i) Modification of existing patterns of feedback by
     a. Removal of some feedback paths
     b. Introduction of new feedback paths
  
  ii) Policy interventions, which have a direct exogenous effect on system behaviour

- The property of an endogenous account of behavioural dynamics. For System Dynamics to be a useful tool the models it produces needs to account for all of the dynamic complexity we see in a system. If dynamic complexity is dominated by exogenous factors, e.g. if the volume of traffic was, for the sake of argument, completely determined by oil price then no amount of System Dynamics modelling would produce further insight into interventions to change traffic volume. System Dynamics modellers aim for endogenous explanation of dynamic complexity where possible.

4. Modelling using System Dynamics

The basic approach to using System Dynamics modelling is shown in Figure 1.

![Figure 1. The System Dynamics methodology adapted from (Sterman, 2000).](image)

The underpinning philosophy behind the approach is based on the idea of double loop learning (Argyris, 1977). Development of a System Dynamics model proceeds through the following stages

1. Development of Causal Loop Diagrams (CLDs). These are used to surface mental models about the behaviour of elements (variables) of the system expressed as causal relationships and feedback loops

2. Stocks and Flows (S&F) Maps. These describe the structure of the system in terms of flows and accumulations of things

3. Model boundary chart. This catalogues all the endogenous, exogenous, and excluded variables from the model

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3 Or avoid modelling engagements in situations where exogenous factors prevail.
4. System Dynamics (SD) Models. These combine CLD+S&F which describe the dynamic behaviour of a system

5. Sub-system diagrams. This provides the overall architecture of a model, comprising sub-systems and flows of things between sub-systems.

The development of CLDs, S&F maps and full System Dynamics models is frequently carried out with the active participation of stakeholders in formalised group model building workshops (Andersen, Vennix, Richardson, & Rouwette, 2007; Vennix, 1996, 1999; Vennix, Andersen, Richardson, & Rohrbaugh, 1992). There is also a role for qualitative uses of System Dynamics where elicitation of feedback loops alone, without recourse to full System Dynamics model development and simulation, is considered by stakeholders sufficient to understand system behaviour and decide interventions (Coyle, 2000). This qualitative use is more properly considered as an example of a class of methods known as Problem Structuring Methods (PSM), which have their own distinctive approach to bringing stakeholders together with a view to systems modelling to help form agreement on interventions (Ackermann, 2012; Eden & Ackermann, 2006; Mingers & Rosenhead, 2004; Rosenhead, 1992, 1996, 2006; White, 2009; Yearworth & White, 2014).

5. Modelling Notation – A Formal Visual Language
The basic concept in a System Dynamics model is that the state of system is self-modifying according to feedback and can be expressed visually as shown in Figure 2.

![Figure 2. Basic concept of System Dynamics – self-modifying state according to feedback.](image)

The box represents a Stock, a quantity of interest in a system that is subject to accumulation, and/or de-accumulation according to the Rate of Flow into/out-of the stock shown by the valve symbol. The cloud symbol indicates the system boundary. This shows that the source or sink of the flow is outside the scope of our system of interest.

The Stock and Flow (S&F) map introduces the meaning of the stock as an integrator of inflows and outflows.

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4 Due to a difference in terminology the System Dynamics community regard a system of systems view as a system and would model it as such i.e. the highest level of view is a system, which can if needed be broken down into sub-system views for convenience of modelling.
The mathematical relationship expressed in the model shown in Figure 3 is given in Appendix B. Note that all System Dynamics models are in effect *integral equation* models of systems, which, conceptually, is different from how engineers are normally taught to model the world. The normal engineering view would be to concentrate on the flows, not the stocks, and express the relationship using differential and partial differential equations. Note that the two views are *equivalent* mathematically, but this conceptual difference can lead to a completely different perspective on problem situations. In fact, Forrester regards this viewpoint as being closer to the way in which the world works and thus offers a lower barrier to achieving a suitable abstraction of a system to model.

The basic notation of Causal Loop Diagrams is shown in Figure 4. A full account of the diagramming convention in System Dynamics modelling and meaning of the links can be found in (Lane, 2008; Schaffernicht, 2010).

Introduction of reinforcing feedback is shown in Figure 5 and shows exponential growth in the state variable. This growth would continue indefinitely but in practice an exogenous factor would come in to play and limit the growth. Conceptually we could think of the source of the thing that is growing being exhausted outside of our system boundary.

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*Equivalent to the point of absurdity where some journals insist on publishing differential equation forms of relationships and System Dynamics modellers have to translate their work accordingly.*
Figure 5. Reinforcing feedback leading to exponential growth. Initial state of the system was 1.0 in 1990 and grew at the fractional growth rate of 5% per annum.

Figure 6 shows the goal seeking behaviour of balancing feedback.

Figure 6. Balancing feedback leading to goal seeking or control behaviour. Initial state of the system was 1.0 in 1990 and the Desired state (goal) was set to 3.0. The Time constant was equal to 10 years.

Figure 7 shows the effect of introducing delay in the feedback loop for various combinations of system Time constant and Delay. The main effect is to introduce oscillation in the system which could be i) damped and eventually lead to convergence on the desired state, or ii) un-damped and lead to divergence where the amplitude in the oscillation grows.
Figure 7. The effect of delays on balancing (controlling) feedback, inducing oscillation due to mismatched time constants.

In a real systems model there is likely to be any number of reinforcing and balancing feedback loops each with characteristic time response and feedback delays. The consequence of their combination leads to the dynamic complexity that, without tools such as System Dynamics to aid in analysis and simulation, would be impossible to understand through intuition alone. The following section illustrates some of the results of System Dynamics modelling analysis of feedbacks in transport systems.

6. Workshop Design

As ever, stakeholder availability limits the quality and quantity of the outcomes from a group model-building workshop. The benefits obtained from the buy-in from actually turning up are balanced by the perceived opportunity cost of attendance. The length of workshops is also critical, balancing the perception of a day lost with the benefits of attending which include actual positive benefit to the day job as well the opportunity for networking. The actual length chosen usually reflects a trade-off. A workshop that is not long enough to achieve full system dynamics modelling with the stakeholders
can focus on eliciting feedback mechanisms from the assembled experts that have not been revealed in either the preliminary analysis or the literature.

6.1. Deciding on Preliminary Modelling

All SD modelling engagements that use group model building are faced with the choice of whether to start from scratch, literally from a blank sheet of paper, or to use a preliminary model based on interviews with stakeholders and/or analysis of relevant documents. This choice is outlined in (Vennix, 1996) and shown in Figure 8. When given an extremely limited amount of time to work with a stakeholder group use of a preliminary model would facilitate a quick introduction to the modelling approach and encourage participants to start modelling as quickly as possible.

![Figure 8. Schema to decide use of preliminary models from (Vennix, 1996).](image)

6.2. Scripts

Scripts set out the detailed planning for a group model building event describing activities undertaken by the group at a task-by-task level. The following example is taken from a recent group model building event held for the Transport Systems Catapult (Yearworth, 2014a, 2014b).
**Part 1: Introduction to SD Modelling**

Mike Yearworth will provide a brief introduction to System Dynamics modelling. This will include something of its origin, uses, methodology, approach and examples. The intention will be to rapidly get the stakeholders up to speed with the notation so that they can *parse* a System Dynamics model and start their own modelling as soon as possible. At the end of this session Mike will make the allocation to sub-groups.

**Part 2: First Exercise – Review and critique of existing models (warm-up)**

As a warm up exercise, Mike Yearworth will introduce a selection of the preliminary models to the stakeholders and ask them to critique them in their sub-groups and suggest improvements. These models will be printed in A1 portrait format to hang on flip charts or whiteboards with one or models for each sub-group. The intention is to familiarise participants with real examples of models and get used to reading them and understanding what they are saying in terms of dynamic behaviour. Feedback loops will be left off to encourage participants to label the models with the appropriate balancing and reinforcing feedback loops. Each sub group will be asked to provide a brief report back to the whole group to get used to presenting models and making sure that everyone in the room sees the whole store of preliminary models.

**Part 3: Second Exercise – ideas for state variables (stocks) of interest to stakeholders**

Having gained some familiarity with the notation Mike Yearworth will then ask the participants to work in their sub-groups to identify possible state variables (stocks) that they feel would be an important quantity to model in a SD model. Mike will circulate around the groups to help if any of the state variables are surfaced as flows, or auxiliary variables. Once a list of state variables has been produced by each group Mike will then ask them pick one at a time and draw them in the form of Figure 3. Mike will supply templates of Figure 3 comprising the basic stock and flow symbols for participants to label. For each stock, Mike will then ask the participants to label the flows and then to think about what factors affect the flows, and how information about the stocks is used (if at all). The intention will be to draw fragments of models, which include at least one stock, inflows and outflows, and relevant causal relationships i.e. what causes the flow rates to change. If any causal loops are identified they will need to be labelled. These model fragments will be presented back to the whole group.

**Part 4: Third Exercise – Auxiliary variables mediating feedback paths from state variables**

This exercise will be a continuation from the second exercise. Having had a chance to hear what other sub-groups have achieved each sub-group will be asked to review their models and develop them further by either i) improving information about causal relationships and possible feedback loops, ii) new state variables missed previously but prompted by hearing what other sub-groups have modelled, and/or iii) discovering connections between models, assembling them together into models that consist of two or more stocks. These will then be presented back to the whole group and represent the sum of the modelling outcomes of the workshop.
7. Mathematical formalism underpinning System Dynamics Models

7.1. Stocks and flows
The basic S&F map in Figure 3 is described mathematically by

\[ S(t) = \int_{t_0}^{t} [\text{Inflow} - \text{Outflow}]dS + S(t_0) \]

The normal engineering view would be to concentrate on the flows, not the stocks, and express the same relationship as

\[ \frac{dS}{dt} = \text{Inflow}(t) - \text{Outflow}(t) \]

Note that the two views are equivalent mathematically. The introduction of reinforcing feedback is shown in Figure 5 and here the mathematical relationships are given by

\[ S(t) = S_0 e^{at} \]

and

\[ \frac{dS}{dT} = gS \]

Figure 6 shows the goal seeking behaviour of balancing feedback. The mathematical relationships are

\[ S(t) = S^* - (S^* - S_0) e^{-t/T} \]

and

\[ \frac{dS}{dT} = (S^* - S)/T \]

Under the hood, System Dynamics models can be simulated by the use of numerical integration techniques based on Euler and Runge-Kutta methods.

7.2. Causal Loops
Causal influences can either be +ve or –ve. If A causes B with +ve influence then, all other things being equal, as A increases (decreases) B increases (decreases) i.e. they move in the same direction. Mathematically this would be described as:

\[ \frac{\partial B}{\partial A} > 0 \]

If X causes Y with -ve influence then, all other things being equal, as X increases (decreases) Y decreases (increases) i.e. they move in the opposite direction. Mathematically this would be described as:

\[ \frac{\partial Y}{\partial X} < 0 \]
References


